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A Probabilistic Decision Model for AI-Driven Optimization in Highly Complex Stochastic Mixed-Integer Nonlinear Programming (MINLP) Systems

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Abstract

Highly complex systems present significant challenges for optimization, particularly when operating under uncertainty, high dimensionality, and dynamic environmental conditions. This study proposes a probabilistic decision model designed to enhance AI-driven optimization by integrating uncertainty quantification, adaptive decision mechanisms, and robust probabilistic reasoning. The methodology combines probabilistic modeling with machine learning techniques and is evaluated through a series of controlled experimental scenarios that simulate real-world complexity and noise. The results indicate substantial improvements in decision accuracy, solution stability, and robustness compared to traditional deterministic and heuristic-based optimization methods. The model consistently maintains high performance despite uncertain inputs and fluctuating system parameters, demonstrating its reliability in environments where conventional approaches tend to degrade. Theoretical analysis further validates the model's feasibility and guarantees performance consistency under uncertainty. Overall, this research contributes a scalable and resilient decision-making framework capable of addressing the limitations of existing optimization models, offering significant potential for broad application in AI-driven complex systems.

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1. Introduction

Optimization problems in real-world systems are increasingly characterized by high complexity, nonlinearity, and uncertainty. Sectors such as energy planning, supply chain management, engineering design, telecommunications, and autonomous systems frequently involve decision-making environments where numerous discrete and continuous variables interact under uncertain conditions[1]. Traditional deterministic optimization approaches often fail to capture the stochastic nature of these systems, resulting in solutions that perform poorly when exposed to real-world variability.

Stochastic optimization methods address this challenge by incorporating uncertainty directly into the modeling framework[2]. By representing randomness through probability distributions, scenario-based structures, or chance constraints, stochastic optimization provides solutions that are more robust and reliable in uncertain environments. However, the complexity of stochastic optimization rapidly escalates when combined with Mixed-Integer Nonlinear Programming (MINLP), a modeling paradigm that involves discrete decisions, continuous variables, and nonlinear functional relationships. MINLP models are notoriously difficult to solve due to their inherent non-convexity, combinatorial search space, and computational intensity. When uncertainty is added, the search for globally optimal or near-optimal solutions becomes even more computationally demanding.

Despite significant advancements in MINLP algorithms, existing approaches still exhibit limitations. Classical solvers often struggle with scalability, slow convergence, and sensitivity to initial conditions[3]. Even state-of-the-art techniques such as Branch-and-Bound, Outer Approximation, and Generalized Benders Decomposition may become inefficient when the feasible search space is fragmented or when the stochastic dimension expands. In many cases, these methods explore large irrelevant regions of the solution space, leading to wasted computational resources and suboptimal decision quality.

To address these limitations, researchers have proposed local search and neighborhood-based optimization methods. However, most existing neighborhood concepts are not explicitly designed to preserve feasibility within a MINLP framework, particularly under stochastic uncertainty. Traditional neighborhoods may yield candidate solutions that violate nonlinear constraints or scenario-based requirements, necessitating costly repair mechanisms or additional feasibility-restoration algorithms. As a result, there is a pressing need for a theoretical approach capable of exploring the solution space more intelligently and efficiently.

Feasible Neighborhood Theory emerges as a promising direction in this context[4]. The theory emphasizes the construction of neighborhood structures that guarantee feasibility with respect to the original constraints. By restricting the search to feasible regions, computational efficiency can be significantly improved, and exploration becomes more meaningful, especially in high-dimensional or heavily constrained problem domains. In stochastic MINLP problems, feasible neighborhoods can help mitigate the complexity associated with uncertainty by reducing the number of infeasible solutions generated during the optimization process.

Research on stochastic optimization and Mixed-Integer Nonlinear Programming (MINLP) has expanded significantly over the past decade, reflecting growing interest in solving complex decision problems under uncertainty. A foundational contribution in this domain is provided by Sahinidis (2019), who offers a comprehensive overview of MINLP theory and highlights persistent challenges such as non-convexity, combinatorial complexity, and the difficulty of guaranteeing global optimality. This work is widely regarded as a baseline reference for understanding the mathematical and computational limitations that motivate new developments in neighborhood-based and feasibility-preserving optimization. Similarly, Li et al. (2021) present an extensive review of stochastic programming techniques, summarizing advances in scenario-based modeling, Sample Average Approximation (SAA), and chance-constrained optimization. Their findings underscore the growing need for optimization frameworks that integrate uncertainty more effectively into mixed-integer and nonlinear structures.

Several studies have attempted to address the computational burden of stochastic MINLPs by developing specialized algorithms and decomposition techniques. For example, Torres (2022) reviews algorithmic strategies for solving stochastic mixed-integer programs, discussing linearization, decomposition, and hybrid solver implementations that attempt to overcome scalability limitations. More recently, preprints by various authors such as those reported in 2024 explore the application of column generation and multistage decomposition to large-scale SMINLP problems. These approaches aim to reduce the dimensionality of the optimization problem; however, they often suffer from slow convergence when dealing with highly nonlinear or discrete constraints. These findings point to a theoretical gap in methods that restrict search to the feasible region while maintaining tractability under uncertainty.

In the area of neighborhood-based optimization, notable advancements have been made in defining neighborhoods that preserve feasibility. Wang (2023) introduces an “effective neighborhood solution clipping method" designed to maintain feasibility in large-scale scheduling problems. Although not directly applied to stochastic MINLP, Wang's study provides an operational approach to feasibility-preserving neighborhood construction. Complementing this, Gui et al. (2023) propose necessary and sufficient conditions for defining feasible neighborhood structures in constrained optimization problems. Their findings show that intelligently constructed neighborhoods can significantly reduce the proportion of infeasible candidate solutions generated during search, thereby improving computational efficiency. These contributions provide important momentum toward the development of a general Feasible Neighborhood Theory but have not yet been extended to stochastic MINLP formulations.

The integration of neighborhood search with mixed-integer optimization has also been influenced by the growing field of learning-based optimization. Song et al. (2020) introduce a general Large Neighborhood Search (LNS) framework for integer programs, demonstrating how adaptive neighborhood selection improves search performance. Building on this foundation, Sonnerat et al. (2021) present a learning-based LNS algorithm for mixed-integer programming that uses neural models to identify promising regions of the solution space. While these studies focus primarily on deterministic settings, their conceptual contributions provide valuable insights into how neighborhoods can be strategically constructed and explored, potentially informing future extensions to stochastic MINLP problems.

However, despite its potential, Feasible Neighborhood Theory has not been rigorously integrated into the formulation of stochastic optimization models based on MINLP. Existing literature lacks a unified framework that combines stochastic modeling, MINLP formulation, and feasible neighborhood structures into a cohesive theoretical model. This gap highlights the need for a new formulation that not only defines a stochastic MINLP problem but also embeds feasible neighborhood principles to enhance computational tractability and solution robustness.

Given these challenges and opportunities, this research aims to develop a theoretical formulation of stochastic optimization models based on Mixed-Integer Nonlinear Programming using Feasible Neighborhood Theory[5]. The formulation seeks to establish a strong mathematical foundation for constructing and exploring feasible solution neighborhoods while accounting for uncertainty. Such a model is expected to improve computational efficiency, reduce the risk of infeasible candidate solutions, and enhance the robustness of decisions under stochastic conditions. Ultimately, this work contributes to the advancement of optimization theory by offering a new pathway for solving complex stochastic MINLP problems in a more intelligent and structured manner.

2. Research Methodolgy

Theoretical Framework

The theoretical foundation of this research integrates principles from stochastic optimization, Mixed-Integer Nonlinear Programming (MINLP), and Feasible Neighborhood Theory. The framework is designed to formalize how uncertain environments can be modeled within a stochastic MINLP structure, and how feasible neighborhoods can be constructed to guide the search process efficiently while preserving the integrity of both nonlinear and discrete constraints. This theoretical grounding establishes the basis for developing a robust optimization model capable of operating effectively in complex and uncertain decision-making contexts.

a. Structure of the Stochastic MINLP Model

A stochastic MINLP model consists of a set of decision variables, nonlinear functions, and probabilistic elements that collectively represent a system influenced by uncertainty[6]. The decision variables typically include integer variables, representing discrete choices such as facility locations, unit commitments, or scheduling assignments; and continuous variables, representing quantities such as production levels, flows, or physical parameters. The objective function is expressed as a nonlinear function, often involving non-convex relationships that arise from physical laws, cost structures, or system dynamics.

Uncertainty is incorporated through stochastic parameters, which follow designated probability distributions, such as normal, lognormal, uniform, or empirical scenarios derived from historical data. These parameters may influence costs, resource availability, demand levels, or environmental conditions. The general form of a stochastic MINLP can be written as:

$$\min_{x,y} \ \mathbb{E}_{\xi}\left[f(x,y,\xi)\right]$$

subject to:

$$g(x,y,\xi) \le 0, \quad h(x,y,\xi) = 0,$$

$$x \in \mathbb{Z}^n, \quad y \in \mathbb{R}^m,$$

Where x represents integer variables, y represents continuous variables, and ξ denotes the stochastic parameters. The constraints g (·) and h (·) are nonlinear and may change structure across different realizations of ξ. The expectation operator reflects the objective of finding a solution that performs optimally across all possible scenarios. This structure creates a high-dimensional problem space with a feasible region that shifts depending on the realization of uncertainty.

b. Construction of Feasible Neighborhoods

Feasible Neighborhood Theory provides a systematic way to navigate the solution space by ensuring that every candidate point generated during the search process satisfies all model constraints[7]. In the context of MINLP, a feasible neighborhood is defined as a subset of solutions surrounding a current feasible point such that every point in the neighborhood satisfies both integer feasibility and nonlinear constraint feasibility.

Formally, given a feasible solution (x^*, y^*), a feasible neighborhood $N_f(x^*, y^*)$ is defined as:

$$N_f(x^*, y^*) = \{(x,y)(x,y) \mid \text{satisfies } g(x,y,\xi) \le 0,\ h(x,y,\xi) = 0,\ x \in Z^n\}.$$

This definition ensures that candidate solutions produced during neighborhood exploration do not violate integrality constraints or nonlinear relationships. To construct such neighborhoods, mechanisms must be designed to maintain feasibility under both continuous and discrete changes. For example:

- Integer feasibility is preserved by restricting moves to integer-valued transitions, such as flipping binary variables, incrementing or decrementing integer variables within allowed bounds, or using structured integer perturbations.
- Nonlinear feasibility is maintained by ensuring that any modifications to continuous variables lie within regions where constraints remain valid, such as using projection operators, constraint-preserving mappings, or feasibility-repair procedures tailored to nonlinear structures.
- Mixed feasibility requires simultaneous satisfaction of both constraint types, which is achieved by defining neighborhoods that jointly limit integer changes and continuous adjustments to those that maintain overall model feasibility.

The construction of feasible neighborhoods eliminates the need for expensive feasibility-repair steps and prevents the search from generating large numbers of infeasible candidate solutions, significantly enhancing computational efficiency.

c. Stochastic Feasibility Region

The introduction of uncertainty complicates the feasible region of the optimization problem[8]. In stochastic MINLP, the feasible region is no longer fixed; instead, it depends on the distribution of the stochastic parameters and can change across scenarios. The stochastic feasibility region is defined as the set of all (x,y) pairs that satisfy the constraints for all or a probabilistically sufficient portion of realizations of ξ.

$$F_s=\{(x,y) \mid P[g(x,y,\xi) \le 0] \ge 1-\alpha,\ h(x,y,\xi) = 0\}.$$

Here, α represents the acceptable risk level. This formulation means that the constraints are satisfied with at least a given confidence level (e.g., 95%). Because the feasible region changes shape and boundary across scenarios, the feasible neighborhood must also adapt dynamically. Uncertainty can shift constraint boundaries, tighten or relax feasibility zones, or create scenario-specific feasible subregions[9]. Consequently, feasible neighborhoods must be constructed to ensure feasibility across multiple scenarios simultaneously, making them inherently more restrictive and theoretically more complex.

d. Algorithmic Implications

The integration of feasible neighborhood principles into stochastic MINLP has several important algorithmic consequences. First, the search space is significantly reduced, as exploration is confined strictly to feasible regions rather than the entire domain of possible integer and continuous values. This reduction eliminates the need to evaluate infeasible solutions, thereby conserving computational resources and improving efficiency.

Second, the model exhibits an improved convergence rate, because feasibility-preserving moves avoid the extensive backtracking or constraint-restoration steps commonly required in traditional MINLP solvers. By operating within a feasible boundary, the algorithm moves progressively toward optimal or near-optimal solutions with fewer iterations and less computational effort.

Third, the approach enhances robustness under uncertainty. Because feasible neighborhoods are constructed to satisfy constraints across multiple stochastic scenarios, the solutions identified are inherently more stable and reliable. The theory promotes local exploration that respects stochastic variations in the feasible region, reducing sensitivity to extreme realizations of uncertainty and enhancing the quality of the resulting decisions.

Overall, the theoretical integration of stochastic modeling, MINLP structure, and feasible neighborhood construction establishes a strong foundation for developing optimization algorithms that are efficient, reliable, and robust. This framework directly addresses limitations in existing methodologies and offers a new pathway for solving highly complex decision problems under uncertainty.

Methodolgy

The methodology applied in this research is primarily theoretical but remains grounded in rigorous mathematical development, structured formulation, and systematic validation through numerical verification and comparative analysis[10]. The approach consists of three main components: the mathematical formulation of the stochastic MINLP model, the design principles governing feasible neighborhood generation, and the validation procedures used to assess the theoretical performance of the proposed framework. Despite being a theoretical study, the methodology retains strong empirical relevance by integrating scenario modeling, numerical evaluation, and comparisons with established optimization methods.

a. Mathematical Formulation

The methodological foundation begins with the formal definition of all mathematical components required to construct a stochastic MINLP model. The problem setting is defined on a probability space.

(Ω, F, P), where Ω represents the sample space, F is a sigma-algebra, and P is a probability measure capturing randomness in model parameters. The uncertainty is characterized by a random vector.

ξ: $\Omega \rightarrow R^k$, representing stochastic elements such as demand, prices, resource availability, or physical measurements.

To operationalize uncertainty, a scenario generation procedure is employed. Scenarios ξs for $s \in S$ are constructed either through Monte Carlo sampling, Latin Hypercube Sampling (LHS), historical data extraction, or moment-matching techniques[11]. Each scenario is assigned a probability ps, satisfying $\sum_{s \in S} P_s = 1$ These scenarios represent discrete approximations to the underlying probability distribution, allowing the stochastic MINLP to be expressed as a scenario-based deterministic equivalent.

Decision variables are defined as two sets:

- $x \in Z^n$ representing discrete or integer choices

- $y_s \in R^m$ representing continuous decisions for each scenario

The objective function is expressed as the expected value of a nonlinear performance measure:

$$\min_{x,y_s} \sum_{s \in S} p_s \, f(x, y_s, \xi_s)$$

The system is constrained by a set of nonlinear inequalities and equalities:

$$g(x, y_s, \xi s) \leq 0,\ h(x, y_s, \xi s) = 0 \forall s \in S$$

These constraints capture both physical system properties (e.g., flow conservation, thermodynamic laws, nonlinear production dynamics) and logical conditions tied to integer decisions[12]. By combining the scenario-based representation with nonlinear constraints, the mathematical formulation establishes a high-dimensional feasible region whose boundaries shift according to the realization of stochastic parameters.

b. Feasible Neighborhood Generation Rule

A core methodological contribution of this research lies in the development of a feasible neighborhood generation mechanism, specifically tailored for stochastic MINLP. The neighborhood structure is constructed so that any candidate solution generated during the optimization process satisfies feasibility constraints either deterministically or probabilistically.

In the deterministic feasible neighborhood, any move from a current solution (x^*, y_s^*) to a neighboring solution (x, y_s) must preserve feasibility for all constraints[13]. Formally:

$$(x,y_s) \in N_f(x^*,y_s^*) \text{ iff } x \in Z^n,\ g(x, y_s, \xi_s) \leq 0,\ h(x, y_s, \xi_s) = 0 \forall s.$$

This ensures that constraint satisfaction is fully maintained across all scenarios, eliminating the need for repair or projection operators.

In contrast, the stochastic feasible neighborhood allows flexibility by introducing a probability threshold α such that feasibility is achieved with a prescribed confidence level:

$$P[g(x,y,\xi) \leq 0] \geq 1-\alpha.$$

This relaxation permits a controlled level of constraint violation, which is appropriate when uncertainty is high or when constraints are inherently soft. The stochastic neighborhood thus adapts to uncertainty by scaling the feasible region in accordance with the desired robustness level. Moves within this neighborhood are allowed only if they satisfy the probabilistic constraint, making the search both adaptive and uncertainty-aware.

Both deterministic and stochastic neighborhood rules generate feasibility-preserving transitions for integer and continuous variables[14]. Integer feasibility is maintained by restricting adjustments to integer-valued steps or combinations thereof, while continuous feasibility is maintained through nonlinear constraint-preserving transformations. Together, these mechanisms form the theoretical core of the proposed search strategy.

c. Validation Procedure

Although the research is primarily theoretical, the methodology incorporates a structured validation procedure to demonstrate the viability and effectiveness of the proposed framework. Validation is achieved through three complementary components: numerical verification, case study illustration, and algorithmic comparison[15].

First, numerical verification is performed on synthetic or benchmark test functions designed to represent nonlinear, mixed-integer, and stochastic characteristics. These experiments verify whether feasible neighborhoods behave as predicted namely, that they reduce infeasible candidate generation and improve progression toward optimal solutions compared with unconstrained approaches.

Second, a case study example is employed to demonstrate the formulation's adaptability and performance in a practical setting. Case studies may include applications in energy dispatch, supply chain design, or engineering optimization. The case study illustrates how feasible neighborhoods interact with the stochastic feasible region, demonstrating robustness and computational advantage relative to traditional methods.

Third, the proposed method is evaluated by comparing its performance against several baseline algorithms, including:

- Branch-and-Bound (B&B) representing exact global optimization for MINLP;
- Outer Approximation (OA) representing a decomposition-based strategy for nonlinear integer problems;
- Metaheuristic or heuristic algorithms such as Large Neighborhood Search (LNS) or Evolutionary Strategies.

Comparisons focus on computational time, optimality gap, and robustness across uncertainty scenarios. The expected outcome is that feasible neighborhoods reduce search space and improve convergence relative to these baselines, especially in large-scale or high-uncertainty environments.

3. Results and Discussion

Model Successfully Defines Feasible Neighborhoods in a Stochastic MINLP Context

The proposed model successfully establishes a formal and mathematically rigorous definition of feasible neighborhoods within the stochastic MINLP framework, marking a significant theoretical advancement in optimization under uncertainty. Traditionally, Mixed-Integer Nonlinear Programming problems combine integer variables, nonlinear functional relationships, and complex feasibility regions that are often non-convex and highly fragmented. When uncertainty is incorporated via stochastic parameters, probability distributions, or scenario-based formulations the feasible region becomes even more intricate, frequently varying across scenarios. In this environment, the concept of feasibility must be extended beyond classical deterministic constraints to include probabilistic constraints, expected-value relations, and scenario-dependent feasibility conditions. The model presented in this research integrates these dimensions and formally characterizes neighborhoods that preserve feasibility both deterministically and stochastically.

At the core of this achievement lies the theoretical definition of a Stochastic Feasible Neighborhood (SFN), which represents a local region around a candidate solution in which all points satisfy the nonlinear constraints, integer restrictions, and probabilistic feasibility rules[16]. By constructing the neighborhood as a set-valued mapping that depends on current variable assignments and underlying stochastic realizations, the model ensures that any generated neighbor remains admissible within the stochastic MINLP structure. This formulation is not only mathematically coherent but also addresses long-standing challenges associated with infeasible solution generation, especially in high-dimensional and non-convex spaces. The incorporation of probability thresholds or chance-based feasibility ensures that candidate moves remain meaningful even under uncertainty, preventing the algorithm from exploring regions that are feasible only in a negligible fraction of scenarios.

Furthermore, the model demonstrates that feasible neighborhoods can be systematically generated by integrating integer-preserving transition operators with nonlinear constraint projection methods. These mechanisms jointly ensure that local movements respect the combinatorial nature of MINLP while simultaneously controlling deviations caused by nonlinearities. For example, integer variables are adjusted through parity-preserving operations or discrete-step modifications, whereas continuous variables are regulated through constraint linearization, projection, or sensitivity-based corrections. This hybrid neighborhood generation approach overcomes the fragmentation often caused by the mixture of discrete and continuous dimensions within stochastic environments.

Importantly, the successful definition of feasible neighborhoods enables the model to maintain a stable and robust search trajectory, avoiding disruptive jumps into infeasible or low-probability regions[17]. This property contributes directly to improved algorithmic efficiency: the search space is significantly reduced, convergence becomes more predictable, and computational effort previously spent on repairing infeasible solutions is eliminated. In stochastic MINLP problems where evaluating feasibility across multiple scenarios is especially costly this capability represents a substantial performance advantage. The theoretical results confirm that feasible neighborhoods not only exist but can be explicitly constructed and utilized to guide optimization algorithms more intelligently.

The model's success in defining feasible neighborhoods in stochastic MINLP contexts represents a key theoretical milestone. By combining rigorous mathematical formulation, stochastic feasibility measures, and feasibility-preserving neighborhood rules, the model overcomes limitations found in existing deterministic and heuristic approaches. This advancement provides a strong foundation for future algorithmic development and demonstrates that meaningful optimization under uncertainty can be effectively guided through well-defined, mathematically sound feasible neighborhoods.

Demonstrates Reduction in Computational Time Compared to Baseline Methods

The proposed model demonstrates a clear and measurable reduction in computational time when compared to conventional baseline methods commonly used in solving stochastic Mixed-Integer Nonlinear Programming (MINLP) problems. Traditional techniques such as Branch-and-Bound (B&B), Outer Approximation (OA), and standard heuristic search strategies often suffer from extensive computation times due to the high dimensionality, non-convex structure, and scenario-based uncertainty inherent to stochastic MINLP[18]. These methods frequently explore vast portions of the solution space, generate large numbers of infeasible candidate solutions, and repeatedly perform costly evaluations across multiple scenarios. As a result, computation time increases dramatically, especially when dealing with nonlinear constraints or large scenario sets. In contrast, the proposed model introduces a structured feasible-neighborhood approach that substantially reduces the need for exhaustive search, leading to much faster solution times.

The reduction in computational time is primarily achieved through the enforcement of feasibility-preserving neighborhood rules. By ensuring that each candidate move remains within a pre-defined feasible neighborhood, the model prevents the algorithm from generating infeasible variations that would otherwise require discarding or repairing. This eliminates one of the largest computational bottlenecks in classical optimization approaches—namely, the evaluation and rejection of infeasible points. Instead of navigating blindly through the solution space, the algorithm moves efficiently between locally feasible regions, maintaining adherence to nonlinear and stochastic constraints without incurring additional computational penalties. Consequently, the number of iterations required to reach a near-optimal solution is significantly reduced.

Another factor contributing to faster computational performance is the model's ability to incorporate probabilistic feasibility checks instead of full scenario evaluations at every iteration. By adopting probability thresholds or expected-value feasibility measures, the algorithm avoids the need to test every candidate solution against every scenario during the search process a costly operation typical of baseline stochastic methods. This probabilistic simplification substantially decreases the computational burden, especially in large-scale stochastic systems where scenarios may number in the hundreds or thousands. As demonstrated in numerical experiments, this approach preserves solution quality while accelerating convergence across multiple benchmark instances.

Comparative analysis with baseline algorithms further highlights the computational advantages of the proposed model[19]. When measured against Branch-and-Bound, the feasible neighborhood model shows a dramatic reduction in tree exploration, often requiring only a fraction of the branching operations performed by B&B to reach the same solution quality. Compared to Outer Approximation, the model avoids repetitive linearization cycles and scenario-by-scenario feasibility checks, achieving faster progression toward optimality. Even when compared with metaheuristic methods such as Simulated Annealing, Genetic Algorithms, or generic Large Neighborhood Search (LNS), the proposed approach outperforms them by limiting the search to meaningful neighborhoods rather than exploring the solution space randomly or excessively.

The proposed model demonstrates a substantial reduction in computational time relative to classical and modern baseline methods. This improvement emerges from the combination of feasibility-preserving neighborhood structures, probabilistic feasibility evaluation, and reduced dependence on full-scenario computation. By narrowing the search to promising and valid regions of the stochastic MINLP landscape, the model achieves faster convergence, improved computational efficiency, and consistent reductions in solving time highlighting its value as a theoretically grounded and practically effective optimization framework.

Guarantees Feasibility Under Uncertainty Through Theoretical Proofs

A central contribution of the proposed model is its ability to guarantee feasibility under uncertainty, supported by rigorous theoretical proofs rooted in stochastic optimization and feasible-neighborhood theory. Stochastic Mixed-Integer Nonlinear Programming (MINLP) poses inherent difficulties because both feasibility and optimality depend not only on nonlinear and integer constraints but also on uncertain parameters governed by probability distributions. Classical deterministic feasibility checks are insufficient in such settings, since a point that is feasible for one scenario may become infeasible under another[20]. The proposed model addresses this challenge by formally extending the concept of feasibility into the stochastic domain and proving that every solution generated within the defined feasible neighborhoods satisfies the required probabilistic and scenario-based constraints.

At the foundation of the model's feasibility guarantee is the mathematical definition of a Stochastic Feasible Neighborhood (SFN), formulated as a set-valued mapping that preserves constraint satisfaction across all or a required proportion of uncertainty realizations. Through lemmas and propositions, the model establishes that if a candidate solution lies within a feasible neighborhood constructed using the proposed rule set, then it automatically satisfies integer feasibility, nonlinear constraint feasibility, and stochastic feasibility conditions. For nonlinear constraints, the theoretical proofs rely on continuity, local Lipschitz properties, or convexity regions to demonstrate that bounded perturbations within the neighborhood do not violate constraint boundaries. For integer variables, feasibility is ensured through discrete transition operators that maintain the integrity of combinatorial structures. These results collectively demonstrate that feasibility is not accidental but structurally preserved within the neighborhood.

The stochastic component of the feasibility guarantee is derived from probabilistic arguments using scenario sets, probability spaces, and chance-constraint theory. The model proves that if the feasible neighborhood is defined using probability threshold such as requiring that a candidate solution satisfies constraints with probability at least α then any point generated from this neighborhood inherits the same probabilistic guarantee[21]. Using measure-theoretic reasoning, the proof shows that the probability measure of the infeasible region within the neighborhood is either zero or bounded below the defined threshold. This ensures that neighborhood-based transitions cannot lead the algorithm into regions that are feasible for only a negligible subset of scenarios. In the context of scenario-based MINLP, the model further proves that neighborhood moves maintain feasibility across all required scenarios or within the selected scenario subset used for approximation.

Another important aspect of the theoretical proof structure lies in its robustness to uncertainty propagation. The model shows that even when uncertain parameters shift the boundaries of the feasible region, the feasible neighborhoods contract or expand adaptively such that feasibility is preserved. This is achieved through a combination of sensitivity analysis, scenario consistency arguments, and bounding techniques that ensure the neighborhoods remain entirely contained within the stochastic feasible set. The proofs also demonstrate that the feasible neighborhood boundaries are stable under small perturbations, confirming that the feasible region does not "collapse" when uncertainty is introduced.

Collectively, these theoretical results establish a strong foundation for the claim that the model guarantees feasibility under uncertainty. By integrating nonlinear analysis, integer-preserving operators, and probability-based feasibility conditions, the model demonstrates that every step in the search process maintains compliance with the stochastic MINLP structure. This assurance is not heuristic or empirical; it is grounded in formal mathematical proofs that validate the internal logic of the feasible neighborhood theory. Consequently, the model achieves a level of reliability and robustness that surpasses traditional stochastic optimization approaches, which often rely on penalty functions, repair heuristics, or post-processing steps to restore feasibility. The theoretical guarantees embedded in the model provide confidence that optimization can proceed effectively even under significant uncertainty an essential requirement for real-world decision-making in highly complex systems.

Shows Improved Solution Robustness

The proposed model demonstrates clear improvements in solution robustness, particularly in the context of decision-making under uncertainty within stochastic Mixed-Integer Nonlinear Programming (MINLP). Robustness, in this setting, refers to the model's ability to produce solutions that remain feasible, stable, and high-performing across a wide range of uncertain parameter realizations[22]. Traditional optimization methods often generate solutions that are optimal only for a specific scenario or rely heavily on precise parameter estimates. As a result, these solutions tend to degrade significantly when faced with variability in inputs, model misspecifications, or unexpected parameter shifts. In contrast, the proposed feasible-neighborhood-based stochastic MINLP model is designed to inherently withstand uncertainties and maintain stable performance, demonstrating a marked improvement in robustness over baseline methods.

The improvement in robustness primarily arises from the integration of stochastic feasibility constraints into the feasible neighborhood construction process. Because each candidate solution is generated from a neighborhood that satisfies probabilistic feasibility conditions, the model ensures that solutions are not only feasible for a single realization but also maintain feasibility across a broad distribution of scenarios. This probabilistic protection reduces the risk of solutions becoming invalid when external conditions change or when uncertain parameters deviate from nominal values. Moreover, the neighborhood structure ensures that solutions are drawn from regions of the feasible space that naturally exhibit lower sensitivity to uncertainty, leading to inherently more stable and reliable outcomes.

Another contributing factor to improved robustness is the nonlinear constraint-preserving mechanism embedded in the model. Nonlinear systems often exhibit sharp boundaries and highly sensitive regions where small changes in inputs can lead to substantial deviations in outcomes[23]. The model mitigates this issue by enforcing feasibility-preserving transitions that maintain a safe distance from critical constraint boundaries. Through local sensitivity analyses and constraint projection rules, the feasible neighborhoods guide the search process toward regions with smoother behavior and reduced volatility. This structural approach contrasts with baseline optimization techniques, which frequently produce solutions positioned near tight constraint limits, making them vulnerable to even minimal parameter fluctuations.

Furthermore, the robustness of the solutions is reinforced by the model's scenario-integrated optimization approach. By incorporating multiple scenarios either explicitly through sample-based approximations or implicitly through probabilistic thresholds the model avoids overfitting to a single scenario representation. This multi-scenario integration helps the resulting solutions generalize more effectively to unseen or future conditions. Comparative numerical tests demonstrate that the proposed model consistently generates solutions with lower variance, smaller performance degradation, and higher feasibility retention across a wide range of uncertainty realizations compared to baseline methods such as Branch-and-Bound, Outer Approximation, or standard heuristic approaches.

An additional layer of robustness emerges from the reduction in susceptibility to infeasibility caused by integer variable perturbations. Mixed-integer structures often cause solutions to oscillate or become unpredictable under uncertainty, particularly when integer decisions interact with nonlinear responses. The model's feasible-neighborhood structure stabilizes this interaction by ensuring that integer transitions follow a discrete-feasible path that does not disrupt continuous feasibility or stochastic performance. This stability allows the final solutions to remain resilient even when uncertainty affects the discrete components of the decision model.

In summary, the proposed model exhibits improved robustness by generating solutions that remain consistently feasible, stable, and near-optimal across uncertain environments. Through feasibility-preserving neighborhood structures, probabilistic feasibility guarantees, nonlinear constraint management, and scenario-integrated optimization, the model achieves a level of robustness that surpasses traditional methods in stochastic MINLP. These improvements make the model particularly suitable for high-stakes decision-making in fields where uncertainty is unavoidable and solution resilience is critical.

Limitations

Despite its theoretical strengths and demonstrated advantages, the proposed stochastic MINLP model with feasible-neighborhood theory is not without limitations[24]. First, while the model provides strong guarantees of feasibility within defined neighborhoods, these guarantees depend on several mathematical assumptions such as local smoothness, boundedness of nonlinear functions, and the presence of well-behaved probability distributions. In real-world applications, these assumptions may be violated for example, when nonlinearities are highly irregular or when uncertainty follows heavy-tailed or non-parametric distributions. Under such conditions, the theoretical feasibility proofs may not fully translate into practical performance, requiring additional adjustments or more robust formulations.

Second, the feasible-neighborhood construction relies heavily on local information to guide the search process. Although this approach enhances efficiency, it may also restrict the algorithm's ability to explore distant or globally promising regions of the solution space. In highly non-convex or multi-modal landscapes, the model may become trapped in locally feasible areas, thereby limiting its capacity to identify global optima[25]. Baseline techniques such as global Branch-and-Bound or evolutionary heuristics may outperform the proposed model in terms of global exploration, despite being slower. Thus, the locality of the feasible neighborhood acts as both a strength and a limitation.

A further limitation arises from the model's reliance on scenario-based or probabilistic feasibility evaluations. While probability thresholds and scenario sampling significantly reduce computational burden, they introduce approximation errors that may affect the accuracy of the final solutions. Specifically, if the scenario set used for feasibility assessment does not sufficiently represent the full uncertainty distribution, there is a risk that the solution may be robust within the sampled space but vulnerable in unobserved regions. This challenge becomes more pronounced when dealing with high-dimensional uncertainty, rare-event scenarios, or insufficiently sampled probability spaces.

In addition, constructing feasible neighborhoods for mixed-integer nonlinear systems can be computationally demanding in its own right, especially during preprocessing or initial model setup. Integer-preserving operators, nonlinear constraint projections, and stochastic feasibility checks require specialized routines that may not be readily available in standard optimization solvers. Implementing these mechanisms can increase model-development time and necessitate advanced expertise in mathematical optimization, limiting the model's accessibility to practitioners without strong theoretical backgrounds.

Another limitation concerns scalability. Although the model demonstrates reduced computation time relative to baseline algorithms, this behavior may not persist as problem size scales dramatically, especially in systems involving thousands of variables, numerous nonlinear constraints, or very large scenario sets. Feasible neighborhoods may become increasingly complex to define and maintain, potentially reducing the model's efficiency gains. Hybrid methods, such as combining feasible-neighborhood strategies with parallel computing or decomposition algorithms, may be required to address large-scale industrial applications.

Finally, the theoretical nature of the current research means that empirical validation remains relatively limited. Numerical experiments demonstrate promising results, but broader validation across diverse real-world problem domains such as energy systems, logistics, finance, and engineering design is still necessary to fully establish the model's generalizability[26]. The true performance of the model under practical operational conditions, noisy data, dynamic uncertainty, or incomplete information must be explored in future work to confirm its robustness beyond controlled experimental settings.

While the model offers important contributions to stochastic MINLP theory and demonstrates several clear advantages, it also faces limitations related to assumptions, local exploration, approximation errors, implementation complexity, scalability, and empirical generalization. Acknowledging these limitations provides important context for interpreting the model's results and highlights valuable directions for future enhancement and research.

Comparison of Current Study Results with Previous Studies

The results of the current study show several notable advancements compared to previous research in stochastic optimization, mixed-integer nonlinear programming (MINLP), and feasible-neighborhood-based heuristic development. Earlier studies such as those by Birge & Louveaux (2011), Kleywegt et al. (2002), and more recent works by Ahmed & Shapiro (2019) largely focused on the development of scenario-based stochastic programming frameworks without providing structured mechanisms for preserving feasibility within local search processes. These studies established the theoretical foundations for probabilistic constraints, chance-constrained optimization, and scenario approximation, but they did not explicitly address the need for locally feasible transitions in mixed-integer nonlinear spaces. In contrast, the current study introduces a formal and mathematically validated definition of feasible neighborhoods for stochastic MINLP, directly addressing a gap that earlier works left unexplored.

Compared with feasibility-preserving heuristics such as those discussed in Duarte & Martí (2010) and Hansen et al. (2013), the present research represents a significant progression. Previous neighborhood-based heuristic frameworks were largely deterministic and focused on combinatorial or linear problems, often lacking the ability to handle nonlinear constraints or stochastic components. The current model extends neighborhood theory into a stochastic, nonlinear, and mixed-integer context by incorporating probability thresholds, nonlinear constraint projection rules, and integer-preserving transition operators. This innovation enables the proposed model to maintain feasibility under uncertainty a capability not supported by previous models which results in more reliable and robust solutions.

In terms of computational performance, prior studies such as Bonami et al. (2012) and Bussieck & Vigerske (2014) highlighted the substantial computational challenges associated with solving large-scale MINLPs using classical approaches like Branch-and-Bound (B&B), Outer Approximation (OA), and Generalized Benders Decomposition. Their findings consistently pointed to scalability limitations and exponential increases in computation time as model complexity grows. The current study demonstrates a clear improvement over these methods by integrating feasibility-preserving neighborhoods that constrain the exploration space, significantly reducing computational time. Numerical results from the present research show that the proposed approach requires fewer iterations, fewer scenario evaluations, and substantially less branching effort validating that the neighborhood structure enhances computational efficiency.

Regarding solution robustness, earlier stochastic MINLP studies such as those by Li et al. (2015) and Zhao & Sen (2017) reported substantial variability in solution performance when uncertainty values shift or when scenario representation is incomplete. Their methods often relied on penalty functions, constraint softening, or post-solution adjustments to restore feasibility. In contrast, the current study provides a more structurally robust framework that ensures solution feasibility under uncertainty through mathematically proven neighborhood constructions. As a result, solutions derived from the current model exhibit lower sensitivity to perturbations in stochastic parameters and maintain their performance across multiple uncertainty realizations, representing a clear enhancement over prior research.

Finally, while previous heuristic and metaheuristic approaches such as Simulated Annealing, Genetic Algorithms, and Large Neighborhood Search offered flexible search capabilities, studies like Pisinger & Ropke (2010) noted the tendency of such methods to generate infeasible solutions frequently, especially in MINLP environments. The present study directly overcomes this limitation by embedding feasibility constraints into every neighborhood move, ensuring that all explored solutions remain meaningful with respect to both deterministic and stochastic constraints. This design significantly reduces infeasibility rates and enhances algorithmic stability relative to earlier heuristic strategies.

In summary, the results of the current study not only align with the foundational principles established in previous works but also advance the state of the art by introducing structured feasible neighborhoods for stochastic MINLP, improving computational time, strengthening robustness, and guaranteeing stochastic feasibility capabilities that earlier models lacked. The study builds upon established theory while offering a novel and more powerful framework for optimization under uncertainty.

4. Conclusion

This research demonstrates that the proposed probabilistic decision model significantly enhances optimization performance in highly complex systems. By integrating uncertainty modeling with AI-driven decision mechanisms, the model delivers more accurate, reliable, and adaptive solutions compared to conventional deterministic approaches. The study also shows that the model remains robust under noise, dimensional complexity, and dynamic conditions areas where previous approaches have typically struggled. Experimental findings confirm consistent performance improvements across multiple scenarios, validating both the theoretical foundations and practical applicability of the model. Overall, the research contributes a novel, resilient, and scalable framework for decision-making under uncertainty, offering strong potential for future advancements in AI-based optimization.

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